MG26018 Simulation Modeling and Analysis

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Fall 2019

Assignment 1

Due Date: October 17 (in class)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show enough intermediate steps.
(c) Write the answers independently.

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Question 1 (5 points)

If $X_1, X_2, \ldots, X_n$ are n independent random variables, and $X_i \sim \text{Exp}(\lambda_i)$, $i = 1, \ldots, n$, prove that

$$\min\{X_1, \ldots, X_n\} \sim \text{Exp}(\lambda_1 + \cdots + \lambda_n).$$

Question 2 (10 points)

Prove $\widehat{L}_Q(T) = \widehat{\lambda}\widehat{W}_Q(T)$ on lecture note Lec2 page 30.2/62. (Hint: Use similar argument for proving $\widehat{L}(T) = \widehat{\lambda}\widehat{W}(T)$ and illustration figure on that page.)

Question 3 (25 points)

Prove Theorem 5 (Limiting Distribution of $M/M/s$ Queue) on lecture note Lec2 page 40/62. (Hint: Use similar argument for proving Theorem 4.)

Question 4 (25 points)

Consider two systems. System 1: There are two *independent* $M/M/1$ queues, each with arrival rate λ and service rate μ. System 2: There is one $M/M/2$ queue with arrival rate 2λ and service rate μ for each server. Which system will perform better (i.e., more efficiently) in terms of the following four measures?

(1) The long-run average number of customers in the system;
(2) The long-run average sojourn time in the system;
(3) The long-run average number of customers waiting in the system;
(4) The long-run average waiting time in the system.

Question 5 (15 points)

Consider an $M/M/1/5$ queue with arrival rate $\lambda = 10$/hour and service rate $\mu = 15$/hour.

(1) What is the probability that an arrival customer finds the station is full?

(2) What is the expected amount of time a customer who enters the station will spend in it?

(3) What is the expected amount of time an arrival customer will spend in the station? (Note: If a customer doesn't enter the station, the amount of time he spends in the station is 0.)

Question 6 (20 points)

Consider such a station. Customers arrive from outside according to a poisson process with rate $\lambda = 10$/hour. There is only one server, and the service time is exponentially distributed with rate $\mu = 15$/hour. When a customer finishes service, with probability 0.2, he will re-enter the station immediately. (For example, he may realize he just forgot to do something.) For this station, find out

(1) The long-run average number of customers in the station;

(2) The long-run average sojourn time in the station. (Caution! When a customer re-enters the station, his previous sojourn time will accumulate.)